VELOCITY FIELD IN THE PRANDTL PROBLEM OF COMPRESSION OF A PLASTIC LAYER

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New velocity fields for the well-known Prandtl solution are found in this paper.

The system of equations of the plane problem of the theory of ideal plasticity with the von Mises fluidity condition is of the form

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau}{\partial y}=0,\quad \frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau}{\partial x}=0; \tag{1}$$

$$(\sigma_x-\sigma_y)^2+4\tau^2=4k^2; \tag{2}$$

$$2\tau\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)=(\sigma_x-\sigma_y)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right); \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{4}$$

where σ_x, σ_y, and τ are the components of the stress tensor, (u, v) is the velocity vector, and k is the yield stress in the case of pure shear.

If a solution of Eqs. (1) and (2) is known, the system (3) and (4) determines a possible velocity field, which is nonunique as a rule. The system (1) and (2) is well studied; a review of the known invariant solutions is given in [1].

One of the most well known and practically important solutions of the system (1) and (2) has been found by Prandtl:

$$\sigma_x=-p-k(x/h-2\sqrt{1-y^2/h^2}),\ \sigma_y=-p-kx/h,\ \tau=ky/h, \tag{5}$$

where p and h are constants. This solution describes the stress state in a plastic layer with thickness 2h compressed by two rigid rough plates.

For the stress field (5) the system (3) and (4) is written in the form

$$y\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)=\sqrt{h^2-y^2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{6}$$

We shall find invariant solutions of the system (6). A group of continuous transformations allowable by the system (6) and found according to [2] is generated by the operators:

$$X_1=\frac{\partial}{\partial x},\quad X_2=x\frac{\partial}{\partial v}-y\frac{\partial}{\partial u},\quad X_3=\frac{\partial}{\partial u},\quad X_4=\frac{\partial}{\partial v},\quad X_5=u\frac{\partial}{\partial u}+v\frac{\partial}{\partial v}.$$

The optimal system of one-parameter subgroups necessary for construction of all significantly different invariant solutions [2] is of the form

$$X_4,\ X_1+\alpha X_5,\ X_2+\alpha X_3,\ X_3+\alpha X_4,\ X_1+\alpha X_2+\beta X_3,$$

where α and β are arbitrary constants. By virtue of the invariance criterion of [2], one can construct invariant solutions only on the subgroups $X_1+\alpha X_5$ and $X_1+\alpha X_2+\beta X_3$. We shall give these solutions.

We shall seek the solution on the subgroup $X_1+\alpha X_2+\beta X_3$ in the form

$$u=-\alpha xy+\beta x+g(y),\quad v=\frac{\alpha}{2}x^2+f(y), \tag{7}$$

where f and g are functions subject to determination from the system (6). Substituting (7) into (6) and solving the system obtained, we have

$$\begin{gathered}u=-\alpha xy+\beta x-\alpha h^2\arcsin(y/h)-\alpha y\sqrt{h^2-y^2}-2\beta\sqrt{h^2-y^2}+C_1,\\ v=\alpha x^2/2+\alpha y^2/2-\beta y+C_2,\end{gathered} \tag{8}$$

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where C_1 and C_2 are arbitrary constants. We note that when $\alpha = 0$ (8) changes into the well-known Nadai solution [3], and when $\alpha \neq 0$ we obtain a new velocity field.

Remark. The condition that the energy dissipation be positive imposes the following constraint on the value of the parameters: $\beta - \alpha y > 0$ if $y \in [-h, h]$.

We shall construct the possible invariant solutions on the subgroup $X_1 + \alpha X_5$. We shall seek a solution invariant with respect to this subgroup in the form

$$u = f(y)e^{\alpha x}, \quad v = g(y)e^{\alpha x}. \tag{9}$$

Substituting (9) into (6), we obtain a system of ordinary differential equations

$$y(\alpha f - g') = (f' + \alpha g)\sqrt{h^2 - y^2}, \quad \alpha f + g' = 0.$$

From this we have

$$\sqrt{h^2 - y^2}g'' + 2\alpha^2 g' y - \alpha^2\sqrt{h^2 - y^2}g = 0.$$

The latter equation reduces by the substitution $g' = gu$ to the Riccati equation

$$u' + u^2 + 2\alpha^2 \frac{y}{\sqrt{h^2 - y^2}} u + \alpha^2 = 0.$$

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STABILITY OF A VISCOPLASTIC RING

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We present a theoretical study of the unsteady deformation of cylindrical metal shells under impulsive loading. We investigate the stability of the inertial motion of the boundaries of a flat ring toward or away from the center under small harmonic perturbations of the boundaries, the velocity, and the stress tensor. We derive a relation for the wave number at which the motion becomes unstable, and compare the result with experimental data.

1. Examples of Modeling Processes. In contrast with articles on the dynamic buckling of cylindrical shells under impulsive external or internal pressures [1-5], we consider problems with large plastic deformations (of the order of 100%). Our method of treating the mechanism of the development of unstable motion is similar to that employed in papers on the instability of motion of a finite mass of liquid with a free boundary [6-8].

Figure 1 shows the result of an experiment on the axisymmetric compression of a D16 Duralumin cylindrical shell by detonation products. The initial outside diameter, wall thickness, and height of the shell were, respectively, 22 × 2.5 × 80 mm. After the experiment the average dimensions were 9.4 × 3.9 × 80 mm with an internal square opening (Fig. 1, magnification 10 ×). In the drawing of a 10 × 2 mm 12Kh1MF steel tube to 6 × 2.2 mm without a mandrel, a square channel is formed (Fig. 2, magnification 10 ×). If we consider another method of longitudinal milling of seamless tubing, namely the reduction of 86 × 10 mm 20 St tubing without a mandrel to 65 × 11 mm in two-roller circular-oval passes, we obtain a square internal channel (Fig. 3).

These examples show that over a wide range of initial deformation parameters of tubes (velocity of boundaries 1-1000 m/sec, mechanical properties of the shell material, etc.) we have a characteristic internal profile. In a number of cases in the drawing and reduction of thick-walled tubes, hexagonal, octagonal, etc. internal channels are formed [9, 10]. Wavy boundaries are formed in the hot drawing of seamless tubes (Fig. 4). Here it is believed

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